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13. ABSTRACT (Maximum 200 words)  A basic formulation for the delamination response of composites has been developed and validated against experimental data. An energy balance is formulated that relates the applied force to interfacial strength under conditions of steady delamination. This observation is validated through numerical simulations and it allows to classify interfaces into hard versus soft categories and may serve as a powerful design tool. A simple formulation that incorporates the effect of rate dependence provides theoretical limits at very low and at very high crack velocities. These limits are also verified experimentally. Finally, a formulation has been developed to predict the post shatter mechanical response of composite plates.		
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## TABLE OF CONTENTS

List of Figures and Appendixes	1
Statement of the problem studied	2
Summary of the most important results	3
List of publications	12
List of participating scientific personnel	13
Bibliography	14
Appendixes	15

## LIST OF FIGURES AND APPENDIXES

### Figures

Figure 1. Schematic of the TCT configuration	6
Figure 2. Comparison of experiment and simulation of a TCT experiment	7
Figure 3. Categorization of interfaces	8
Figure 4. Rate dependence in force-displacement law	9
Figure 5. Rate dependence in measured fracture energy	10
Figure 6. Verification of the interface element	11

### Appendixes

Appendix A. Rate independent analysis of TCT experiment	15
Appendix B. Rate dependent analysis of TCT experiment	17

## **STATEMENT OF THE PROBLEM STUDIED**

Limited design tools and design data is available for the design of interfaces subjected to mechanical loading. The basic modes of delamination have not been studied thoroughly to provide a tool with predictive capability, especially for safety applications. The object of this project is to study the most common mode of delamination, viz., under tensile loading. A simple model is developed that can predict the delamination response and mechanical failure of an interface under tension. This model is refined to study the effects of time-dependence of the laminate material on the delamination response. Ultimately, the aim is to incorporate the delamination model into an interface model that can be used to predict post-fragmentation behavior of composite plates.

## SUMMARY OF THE MOST IMPORTANT RESULTS

Layered structures, especially polymer composites, are employed in several critical applications including aircraft and automotive structures, containers for hazardous and sensitive materials, and micro-electromechanical devices. Knowledge of the failure behavior of composite materials is essential in the design of these structures. Delamination is one of the major reasons for the failure of structures employing composite materials. Data on delamination in composite materials is scarce when available, and is expensive to obtain through experiments. It is, therefore, essential to develop numerical simulation techniques that can be used to study and predict physical phenomena caused by delamination in composite materials. This STIR project is focused on building a tool that can reliably predict the interfacial delamination response in practical engineering applications, and validating this tool with experimental data. The approach of this study is to develop numerical formulations for simulating several test models. Wherever possible, theoretical limits are calculated analytically to provide comparisons with simulations as well as experiments.

The simplest form of mechanical test that may be devised to study delamination in a composite material is an arrangement where the rigid substrate of the composite material is cracked and the laminate is allowed to debond from the substrate. This configuration is called the *Through-Cracked-Tension* (TCT) specimen and is shown schematically in Figure 1. A glass-polymer-glass composite specimen was employed in the experiments for the present study. The TCT specimen consists of a glass-polymer-glass laminate system with a through crack in the glass sheets. This composite is used extensively in the automotive industry for automobile windshields and in the construction industry for architectural glass. The interlayer, *polyvinylbutyral* (PVB) in the present case, holds the glass fragments together in the case of shattering due to a collision and absorbs energy upon impact even after the fragmentation of glass. The TCT model provides a fundamental understanding of the delamination process in a glass-polymer-glass system. It may be used to estimate interfacial properties as well as to define constitutive models that govern failure of polymer ligaments connecting glass fragments such as in post shatter behavior of automotive windshields.

Under conditions of steady delamination, a basic energy balance may be employed to relate the applied force to the interfacial strength. The steady state force, measured during an experiment, can be used to extract the interfacial adhesion strength. This analytical result is summarized in Appendix A. The main observation from this formulation is that the force reaches a constant

value under steady state conditions. This observation is also confirmed experimentally. The experiment is simulated using cohesive finite elements for the glass-polymer interface, regular finite elements for the polymer layer, and rigid finite elements for glass. Cohesive elements are finite elements based on cohesive formulations [see, for example, Xu and Needleman (1994), Rahulkumar et al. (1998)], that allow surfaces to separate under a specified traction-displacement law. Usually, this traction-displacement law is derived from a potential function in order to make the work of fracture path-independent. The polymer was modeled as a hyperelastic material, thus accounting for large strains, but ignoring rate effects.

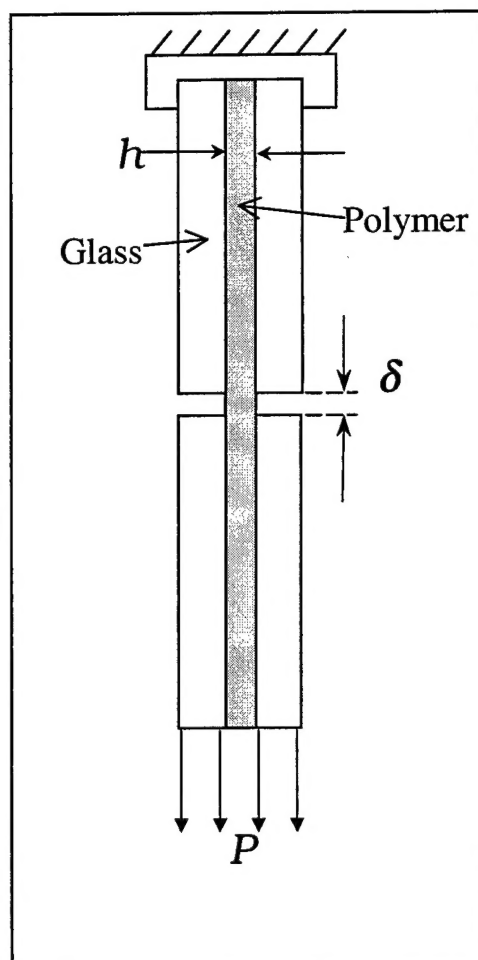
A comparison of the simulation using cohesive elements and the experimental data is shown in Figure 2. This validates the numerical implementation of the cohesive formulation through cohesive elements. The analytical results can be manipulated to obtain a relation between strain at steady delamination and the interfacial adhesion strength. This relation, shown graphically in Figure 3, is used to characterize interfaces as *hard* and *soft*.

To study the effect of rate-dependence of the polymer on the delamination response, the polymer was modeled as a linear-viscoelastic solid. The equation of equilibrium for a moving crack can be solved to obtain the steady state force as a function of the crack velocity. The details of this formulation may be found in Appendix B. The effect of rate-dependence of the polymer on the force-displacement response is shown in Figure 4. Even though the intrinsic fracture energy remains unchanged, the measured fracture energy increases with increasing rates. This observation is summarized in Figure 5, where measured fracture energy is shown as a function of crack tip velocity. These results give an estimate of the range of velocities where rate effects are important. The limiting values approached at very high and at very low velocities are consistent with scaling laws.

The results obtained for a TCT specimen are employed to formulate a model for the constitutive behavior of the polymer ligaments that bridge glass fragments. The delamination response of a TCT specimen is used as the traction-displacement law for normal opening in an interface finite element that replaces the debonded polymer ligament in a numerical simulation. The shear and bending stiffnesses for this element are provided based on the shear and bending stiffnesses of the delaminated polymer. As a validation, this interface element was used to reproduce the behavior of an uncracked monolithic elastic plate under large deformations, by making the polymer layer sufficiently stiff. These validation results are shown in Figure 6. Further refinement of this interface element and validation with experiments is under progress.

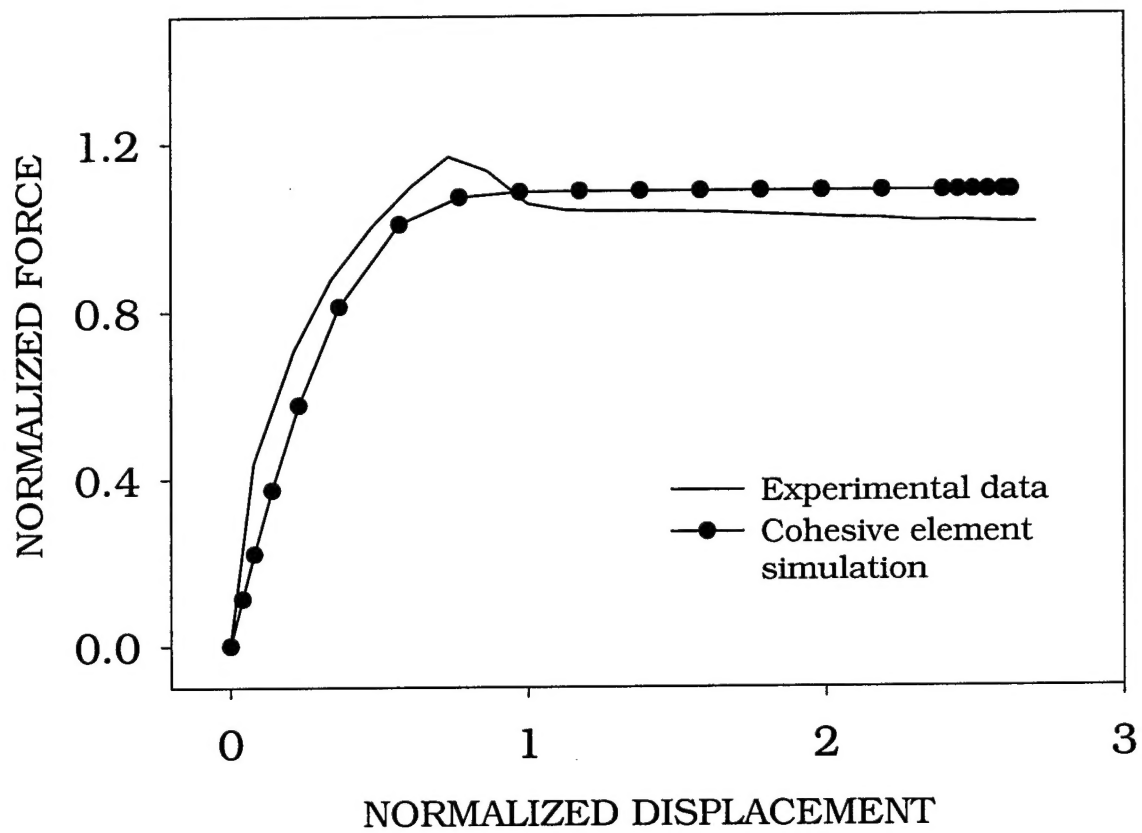
To summarize, the following have been achieved in the present STIR effort:

- (a) a basic formulation for the delamination response of composites has been developed and validated against experimental data,
- (b) a simple model to study the effect of rate dependence has been developed and validated with theoretical limits at very low and at very high crack velocities, and
- (c) a formulation has been developed to predict the post shatter mechanical response of composite plates.

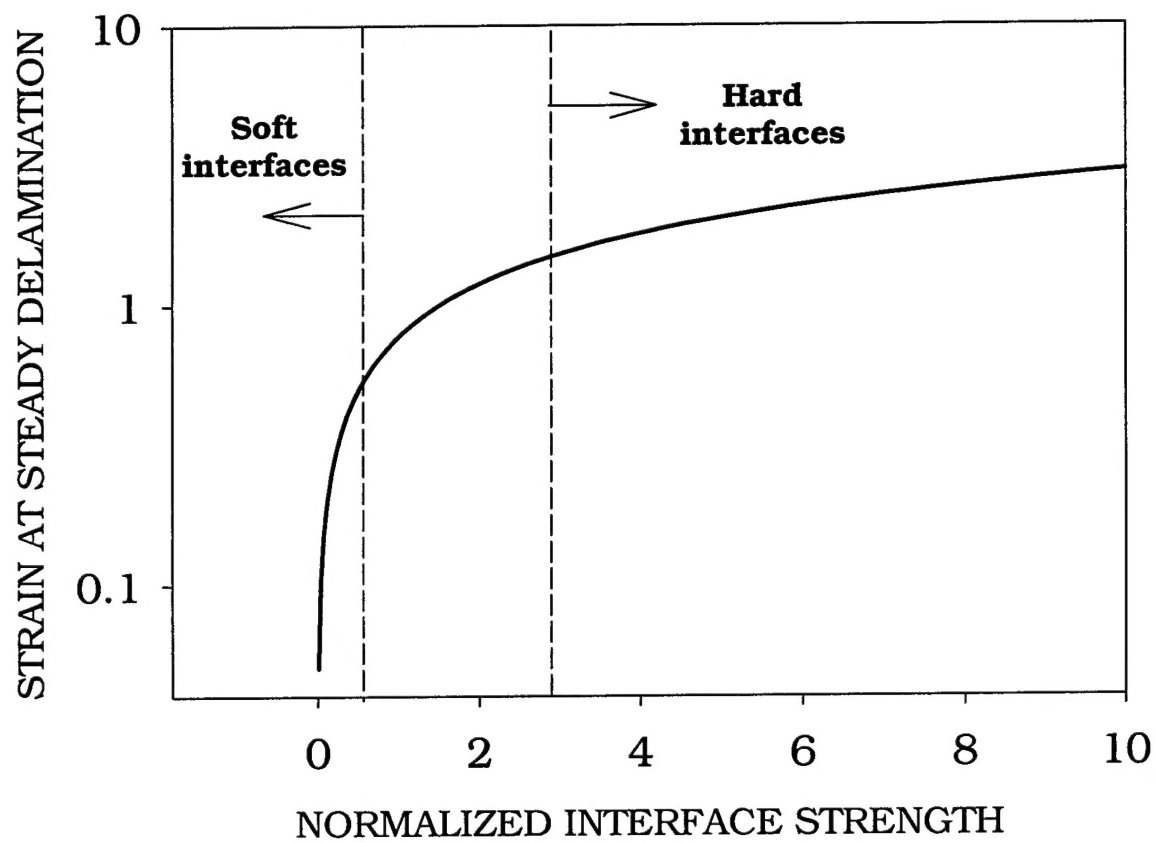


**Figure 1. Schematic of the TCT configuration**

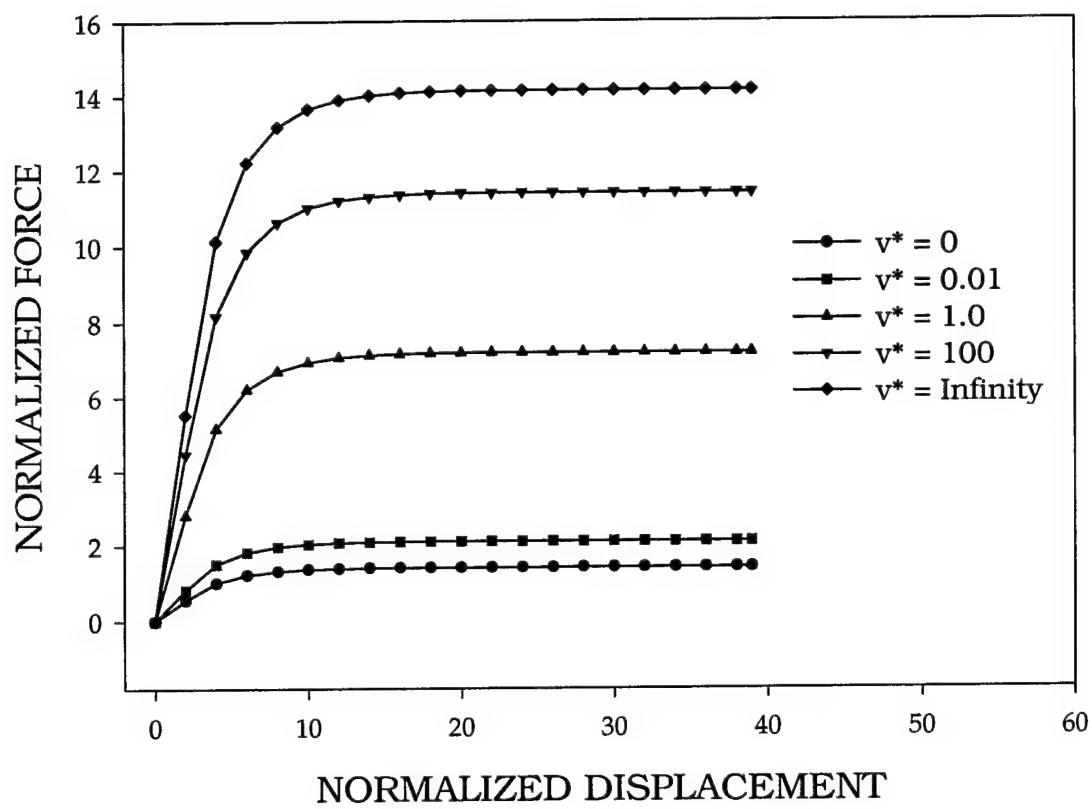




**Figure 2. Comparison of experiment and simulation**

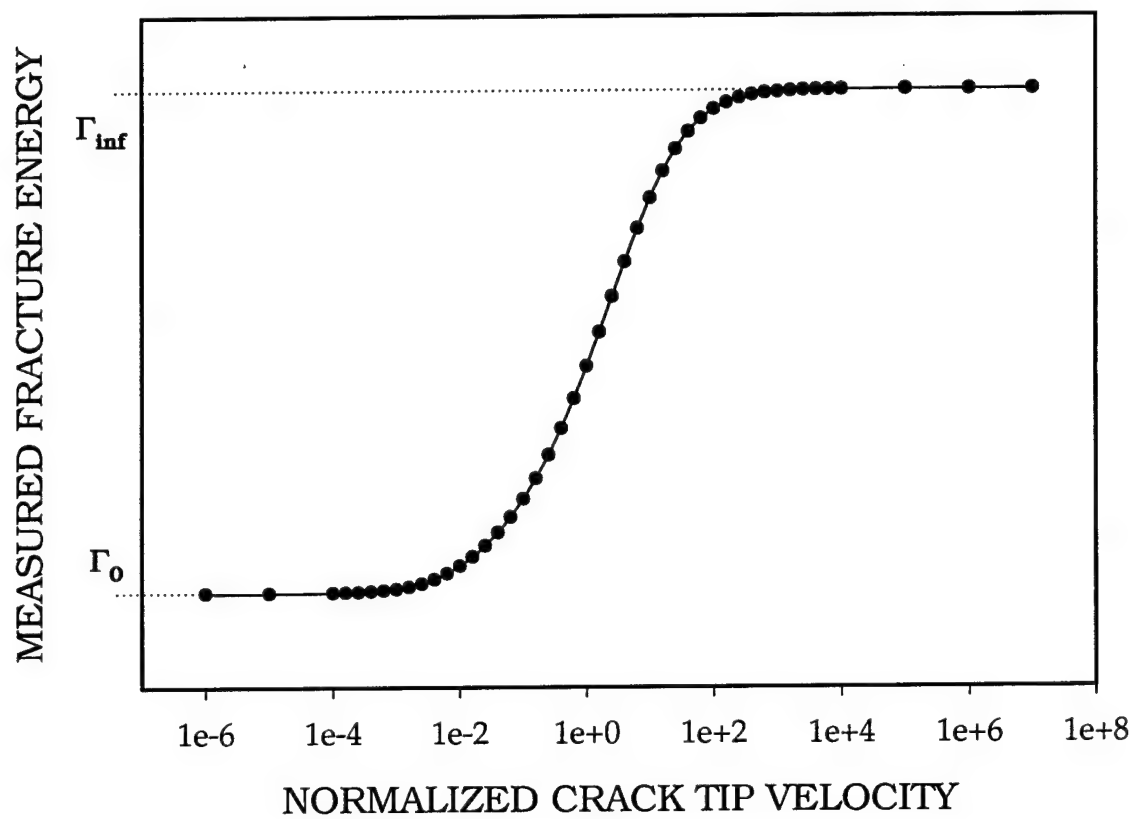


**Figure 3. Categorization of interfaces**



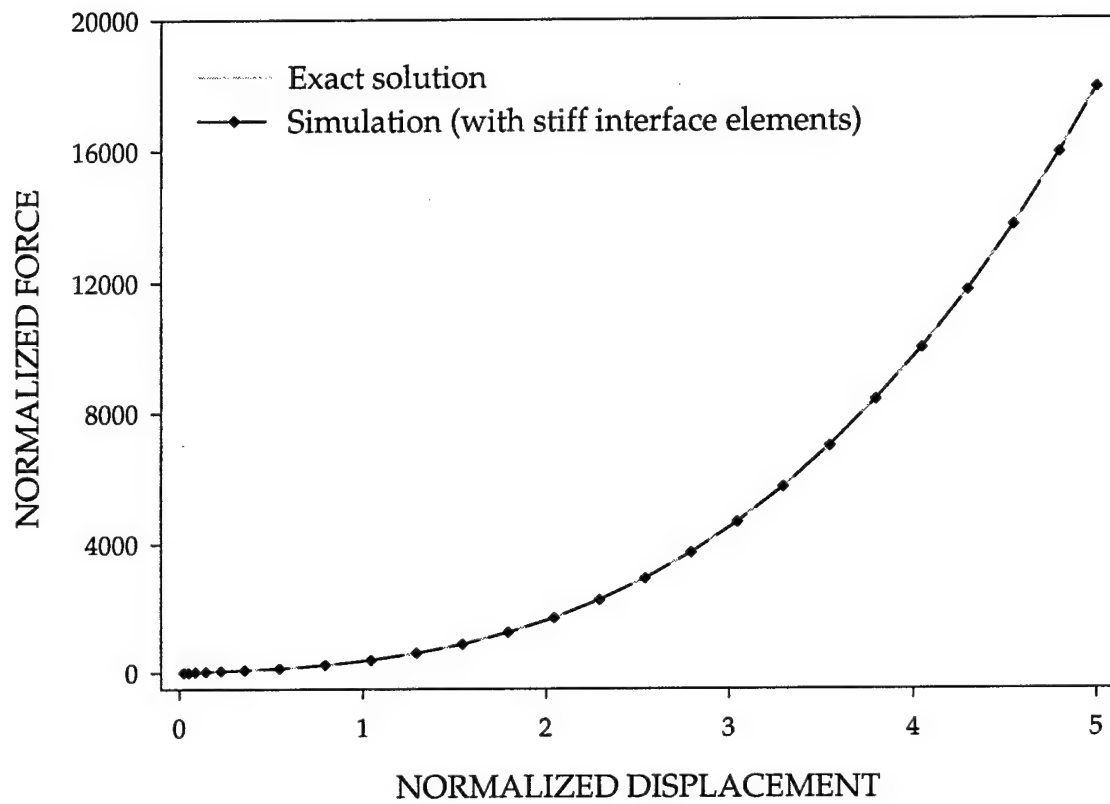
**Figure 4. Rate dependence in force-displacement law**

( $v^*$  : Normalized crack tip velocity)



**Figure 5. Rate dependence of measured fracture energy**

( $\Gamma_0$ ,  $\Gamma_{inf}$  : Limiting interfacial fracture energies at very low and very high strain rates, respectively)



**Figure 6. Verification of interface element**

## LIST OF PUBLICATIONS

S.Muralidhar, A.Jagota, S.J.Bennison, S.Saigal, "Mechanical Behavior in Tension of Cracked Glass Bridged by an Elastomeric Ligament", *Acta Materialia*, submitted.

## **LIST OF PARTICIPATING SCIENTIFIC PERSONNEL**

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S. Jagota, E.I. DuPont de Nemours Inc.

S.J. Bennison, E.I. DuPont de Nemours Inc.

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P.Rahulkumar, A.Jagota, S.J.Bennison, S.Saigal, "Cohesive element modeling of viscoelastic fracture: application to peel testing of polymers", *International Journal of Solids and Structures* 37 (2000) 1873-1897.

Xu, X.P., Needleman, A., "Numerical simulations of fast crack growth in brittle solids", *Journal of Mechanics and Physics of Solids* 42 (1994) 1397-1434.



## APPENDIX A. RATE INDEPENDENT ANALYSIS OF TCT EXPERIMENT

Let  $S$  denote the Piola-Kirchhoff stress in the direction of crack propagation. Let  $\tau(X)$  denote the loading on the crack face. The statement of equilibrium can be written as

$$\frac{dS}{dX} - \frac{\tau(X)}{h} = 0$$

Consider a hyperelastic material with a potential  $U(\epsilon)$  so that the stress is given by

$$S = \frac{dU}{d\epsilon}$$

Let the cohesive surface tractions be given by a potential  $\Phi(\delta)$  as

$$\tau = \frac{d\Phi}{d\delta}$$

The derivative with respect to location  $X$  can be replaced by a derivative with respect to the displacement  $\delta$  to obtain

$$\epsilon \frac{dS}{d\delta} = \frac{1}{h} \frac{d\Phi}{d\delta}$$

or, in differential form

$$h\epsilon dS = d\Phi$$

The left-hand side appears as a differential in the complimentary strain energy expression as

$$\int_0^S \epsilon dS \equiv U^c(\epsilon) = S\epsilon - U(\epsilon)$$

At the cohesive zone tip  $X=0$ , the displacement, stress, and strain all vanish. Therefore, integrating both sides between the cohesive zone tip and a location  $X$ ,

$$hU^c(\epsilon) = \Phi(\delta) - \Phi(0)$$

As  $\delta$  approaches infinity,  $\epsilon$  approaches  $\epsilon_{ss}$  and  $\Phi(\delta) - \Phi(0)$  approaches  $\Gamma_0$ . Therefore,

$$hU^c(\epsilon_{ss}) = \Gamma_0$$

For an elastic material, this result can be written as

$$\frac{1}{2} E h \epsilon_{ss}^2 = \Gamma_0$$

Or,

$$\epsilon_{ss} = \sqrt{\frac{2\Gamma_0}{Eh}}$$

The stress (normalized force) then relates to the fracture energy through the relation

$$\sigma_{ss} = \sqrt{\frac{2E\Gamma_0}{h}}$$

## APPENDIX B. RATE DEPENDENT ANALYSIS OF THE TCT EXPERIMENT

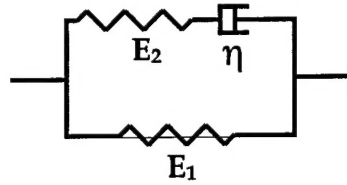
Let  $S$  denote the Piola-Kirchoff stress in the direction of crack propagation and  $\tau(X)$  the loading on the crack face. The statement of equilibrium can be written as

$$\frac{dS}{dX} - \frac{\tau(X)}{H} = 0$$

Let the locations and displacements be non-dimensionalized by the polymer thickness  $H$ , and the stresses by the modulus  $E_1$ . Then,

$$\frac{dS^*}{dX^*} = \tau^*$$

Consider a linear viscoelastic solid (Kelvin) material.



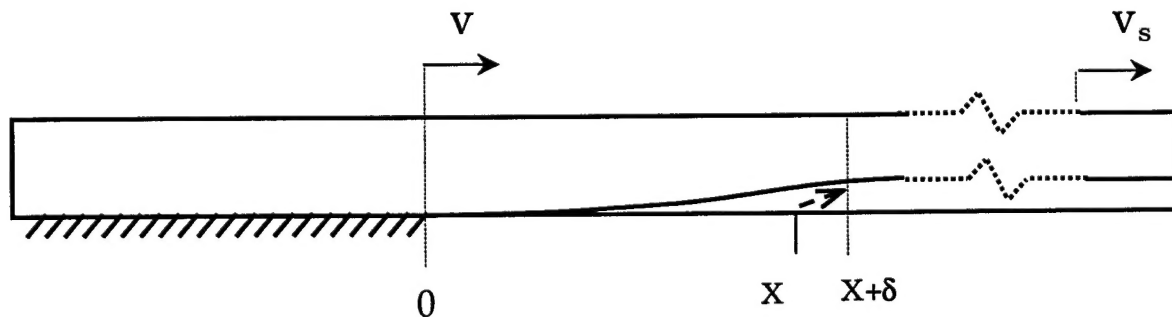
The stress is related to the strain through the differential operator as

$$\left(1 + \frac{\eta}{E_2} \frac{D}{Dt}\right) \frac{S}{E_1} = \left(1 + \left[\frac{\eta}{E_1} + \frac{\eta}{E_2}\right] \frac{D}{Dt}\right) \epsilon$$

At steady state, the material time derivative of any scalar quantity  $A$  can be replaced by the space derivative as

$$\frac{DA}{Dt} = v \frac{dA}{dX}$$

At steady state, the velocity of any material point is proportional to the stretch at that



point. This can be deduced by mass balance

$$\frac{d}{dX} \left( \rho B \frac{H}{\lambda} v \right) = 0$$

Since  $\rho$  (mass density),  $B$  (width) and  $H$  (thickness) are constants,  $(v/\lambda)$  is also constant. At  $X=0$ ,  $l=1$  and  $v=v_0$ . Therefore,

$$v(X) = v_0 \lambda(X)$$

The stress can be expressed, from the equilibrium equations as

$$S^* = \int_0^{x^*} \tau^*(X^*) dX^*$$

Thus,

$$\left( 1 + \frac{\eta v_0}{E_2 H} \lambda \frac{d}{dX^*} \right) \int_0^{x^*} \tau^*(X^*) dX^* = \left( 1 + \left[ \frac{\eta v_0}{E_2 H} + \frac{\eta v_0}{E_1 H} \right] \lambda \frac{d}{dX^*} \right) \varepsilon$$

Or

$$\varepsilon + \alpha \lambda \frac{d\varepsilon}{dX^*} = \int_0^{x^*} \tau^*(X^*) dX^* + \beta \lambda \tau^*$$

where

$$\alpha = \left[ \frac{\eta v_0}{E_2 H} + \frac{\eta v_0}{E_1 H} \right]$$

$$\beta = \frac{\eta v_0}{E_2 H}$$

Differentiating throughout by  $X^*$  and rearranging,

$$\frac{d}{dX^*} \left( \varepsilon + \alpha \lambda \frac{d\varepsilon}{dX^*} - \beta \lambda \tau^* \right) - \tau^* = 0$$

Replacing the derivative with respect to  $X^*$  by that with respect to  $\delta$ ,

$$\varepsilon \frac{d}{d\delta^*} \left( \varepsilon + \alpha \lambda \varepsilon \frac{d\varepsilon}{d\delta^*} - \beta \lambda \tau^* \right) - \tau^* = 0$$

Or

$$\varepsilon \frac{d}{d\delta^*} \left[ \varepsilon + (1 + \varepsilon) \left( \alpha \varepsilon \frac{d\varepsilon}{d\delta^*} - \beta \tau^* \right) \right] - \tau^* = 0$$

The above equation can be solved numerically to obtain the stress (which is the normalized force) as a function of the crack-tip velocity (inherent in the parameters  $\alpha$  and  $\beta$ ).